THERMAL FLUX DISTRIBUTION OVER THE INSIDE SURFACE OF A HEATED TUBE WITH A CYLINDRICAL GROOVE ON THE OUTSIDE

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The heat conduction problem is solved for an electrically heated tube with a cylindrical groove on the outside surface. The thermal flux over the inside surface is calculated as a function of the geometrical parameters.

In order to vary the thermal flux over the length of a heated tube in hydraulic and heat-transfer experiments with a boiling liquid in heated tubes, one often resorts to varying the thickness of the heat-conducting wall. Thus, for example, a local heat riser is simulated by cutting a cylindrical groove around the outside surface of a tube [1, 2].

In evaluating the test results, it is usually assumed that the thermal flux changes at the groove stepwise, i.e., the lengthwise heat leakage along the tube wall is not taken into account.

In this article we will analytically calculate the thermal fluxes transmitted to a boiling liquid at the groove region. From these results we can then estimate the maximum thermal flux under a groove as well as the variation of the thermal flux along the tube.

We consider an infinitely long tube with an inside radius R_0 and an outside radius R_1, turned down to a radius R_2 ($R_0 < R_2 < R_1$) over a tube segment of a length 2L. The outside surface of the tube is thermally insulated, while the inside surface is maintained at a constant temperature T_0. It is assumed that the volume rate of heat generation, which is constant across a wall section, changes stepwise at the groove edge. It is also assumed that the thermal conductivity λ of the material does not depend on the temperature.

With the z-coordinate along the tube axis and the origin of coordinates at the groove edge, we have the following equations for the temperature field in the wall:

$$\frac{1}{R}\cdot\frac{\partial}{\partial R}\left(R\frac{\partial T_i}{\partial R}\right)+\frac{\partial^2 T_i}{\partial Z^2}=-\frac{q_{vi}}{\lambda}, \tag{1}$$

where i = 1 for the seamless tube and i = 2 for the grooved segment.

After a change to dimensionless variables according to the formulas

$$r=\frac{R}{R_1-R_0},\quad Z=\frac{z}{R_1-R_0},\quad \theta=\frac{(T-T_0)\lambda}{q_{v1}(R_1-R_0)^2} \tag{2}$$

Eqs. (1) become for the seamless tube

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial\theta_1}{\partial r}\right)+\frac{\partial^2\theta_1}{\partial Z^2}=-1, \tag{3}$$

and for the grooved segment

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial\theta_2}{\partial r}\right)+\frac{\partial^2\theta_2}{\partial Z^2}=-\varepsilon^2, \tag{4}$$

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where

$$\varepsilon^2 = \frac{q_{v2}}{q_{v1}} \cdot \frac{R_2^2 - R_0^2}{R_1^2 - R_0^2}. \tag{5}$$

Considering that the quantity of heat generated per unit volume is proportional to the electric current density squared at a given section, we obtain from (2) and (5) the following expression

$$\varepsilon = \frac{r_1^2 - r_0^2}{r_2^2 - r_0^2}, \tag{6}$$

for the parameter ε, which is equal to the ratio of asymptotic thermal fluxes at the inside surface under a groove (in the case of an infinitely long grooved segments) and at the inside of a seamless tube, respectively.

Taking advantage of the symmetry in this problem, symmetry with respect to the groove center, we write the boundary conditions for Eqs. (3) and (4) as

$$\begin{gathered} \theta_1|_{r=r_0} = \theta_2|_{r=r_0} = 0 \quad \left.\frac{\partial\theta_1}{\partial r}\right|_{r=r_1} = \left.\frac{\partial\theta_2}{\partial r}\right|_{r=r_2} = 0, \\ \left.\frac{\partial\theta_1}{\partial Z}\right|_{Z\to 0} \to 0, \quad \left.\frac{\partial\theta_2}{\partial z}\right|_{Z=l} = 0, \end{gathered} \tag{7}$$

where

$$l = \frac{L}{R_1 - R_0}.$$

The condition that the temperatures must be equal and the thermal fluxes must be equal at the groove boundary yields

$$\theta_1|_{Z=0} = \theta_2|_{Z=0}, \quad \left.\frac{\partial\theta_1}{\partial Z}\right|_{Z=0} = \left.\frac{\partial\theta_2}{\partial Z}\right|_{Z=0} \quad r_0 \leqslant r \leqslant r_2, \tag{8}$$

$$\left.\frac{\partial\theta_1}{\partial Z}\right|_{Z=0} = 0 \quad r_2 < r \leqslant r_1. \tag{9}$$

A solution to the problem thus stated will be sought in the form of a series in eigenfunctions which satisfy the following relations:

$$\frac{1}{r} \cdot \frac{d}{dr}\left(r\,\frac{du_{in}}{dr}\right) + \mu_{in}^2 u_{in} = 0, \quad u_{in}|_{r=r_0} = 0, \quad \left.\frac{du_{in}}{dr}\right|_{r=r_i} = 0, \tag{10}$$

where i = 1 for the seamless tube segment and i = 2 for the grooved segment; the eigenfunctions of problem (10) are expressed in terms of Bessel functions of the first and the second kind:

$$\begin{gathered} u_{in} = J_0(\mu_{in} r)\, Y_1(\mu_{in} r_i) + Y_0(\mu_{in} r)\, J_1(\mu_{in} r_i) \\ i = 1,\ 2, \quad n = 1,\ 2,\ 3,\ \ldots, \end{gathered} \tag{11}$$

while the eigenvalues μ_{in} satisfy the equation

$$J_0(\mu_{in} r_0)\, Y_1(\mu_{in} r_i) - J_1(\mu_{in} r_i)\, Y_0(\mu_{in} r_0) = 0. \tag{12}$$

If the thermal flux at the groove edge is represented by a series of eigenfunctions u_{1n} with undetermined coefficients a_n,

$$\left.\frac{\partial\theta_1}{\partial Z}\right|_{Z=0} = \begin{cases} \sum\limits_{n=1}^{\infty} a_n u_{1n}, & r_0 \leqslant r \leqslant r_2, \\ 0, & r_2 \leqslant r \leqslant r_1, \end{cases} \tag{13}$$

then, by virtue of conditions (8)-(9) and with the consideration of the last two equations in (7), we obtain the following infinite system of equations for determining the coefficients a_n:

$$\sum_{j=1}^{\infty} a_j \sum_{n=1}^{\infty} \frac{1}{\mu_{1n}\|u_{1n}\|^2} \int_{r_0}^{r_2} u_{2K} u_{1n} r\,dr \int_{r_0}^{r_2} u_{2j} u_{1n} r\,dr + a_K \frac{\|u_{2K}\|^2 \operatorname{cth}(\mu_{2K} l)}{\mu_{2K}} = \frac{k_{1K}}{\mu_{2K}^2} - \sum_{n=1}^{\infty} \frac{\varepsilon^2 k_{2K}}{\mu_{1n}^2 \|u_{1n}\|^2} \int_{r_0}^{r_2} u_{2K} u_{1n} r\,dr, \tag{14}$$

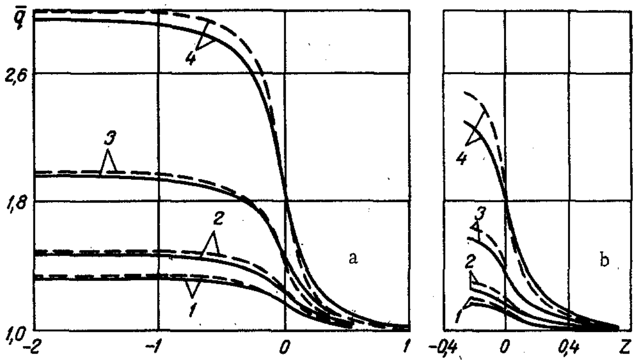

Fig. 1. Distribution of relative thermal flux $\bar{q}$ over the inside surface of a tube within the groove region. a) l = 0.5; b) l = 4: 1) ε = 1.35; 2) 1.5; 3) 2.0; 4) 3.0. Solid curves refer to r_0 = 1; dashed curves refer to r_0 = 9.

$$k = 1,\ 2,\ 3,\ \ldots,$$

where

$$\|u_{in}\|^2 = \int_{r_0}^{r_i} u_{in}^2 r dr, \quad k_{1n} = \int_{r_0}^{r_2} u_{2n} r dr, \quad k_{2n} = \int_{r_0}^{r_2} u_{1n} r dr, \tag{15}$$

and the expression for the dimensionless temperature become

$$\theta_1 = -\sum_{n=1}^{\infty}\left[\exp(-\mu_{1n}Z)\sum_{j=1}^{\infty} a_j \frac{r_2 u_{2j}}{\mu_{2j}^2 - \mu_{1n}^2}\left.\frac{du_{1n}}{dr}\right|_{r=r_2} + \frac{\varepsilon^2 k_{2n}}{\mu_{1n}}\right]\frac{u_{1n}}{\mu_{1n}\|u_{1n}\|^2}, \tag{16}$$

$$\theta_2 = -\sum_{n=1}^{\infty}\left\{-a_n\|u_{2n}\|^2\frac{\operatorname{ch}[\mu_{2n}(l-Z)]}{\operatorname{sh}(\mu_{2n}l)} + \frac{k_{1n}}{\mu_{2n}}\right\}\frac{u_{2n}}{\mu_{2n}\|u_{2n}\|^2}. \tag{17}$$

Differentiating (16) and (17) with respect to r and inserting $r = r_0$ will yield expressions for the dimensionless thermal fluxes at the inside of a tube.

This problem was solved numerically on a computer. The eigenvalues were found from Eq. (12) by the method of steepest decent, within an absolute accuracy of 10^{-6}. In series (13) we retained a finite number of terms, and then reduced system (14) to a finite system of equations linear with respect to the unknown coefficients a_n.

The results shown here have been obtained by retaining the first 10 terms of series (13), sufficient to hold the maximum error in the calculated thermal flux at the inside surface within 1%, as has been verified by sample calculations with double the number of retained terms.

The calculated results were plotted in terms of the relative thermal flux given by the expression:

$$\bar{q} = \frac{q}{q_{1\infty}} = \frac{2r_0}{r_0 + r_1}\cdot\left.\frac{\partial\theta}{\partial r}\right|_{r=r_0}, \tag{18}$$

with $q_{1\infty}$ denoting the asymptotic thermal flux at a sufficiently far distance from the groove.

In Fig. 1 we show the calculated longitudinal distributions of thermal flux over the inside surface of a tube with a narrow groove (l = 0.5) and with a wide groove (l = 4) for various values of ε.

It can be seen in Fig. 1 that the thermal flux varies smoothly from its maximum at the groove over a segment length equal to double the tube wall thickness (within $-0.5 \le z \le 0.5$) and that the slope of the curves increases as the groove becomes less cylindrical, i.e., as r_0 increases. According to Fig. 2, where the calculated maximum thermal flux over a groove is shown as a function of the groove width at various values of r_0 and ε, the maximum thermal flux over a grooved segment may differ considerably from the asymptotic thermal flux for an infinitely wide groove.

Already when the groove width is equal to double the wall thickness, the maximum thermal flux decreases occasionally to less than 10% below the theoretical value which does not account for lengthwise heat leakage.

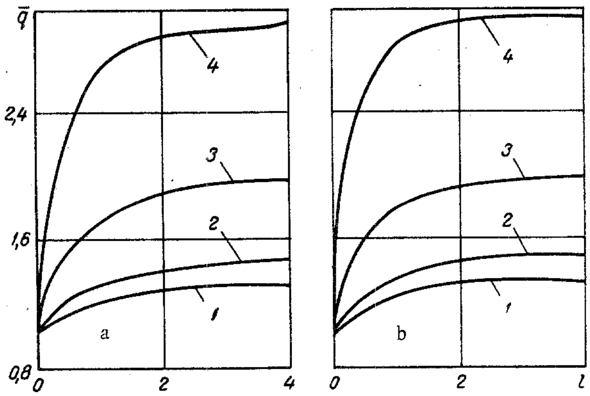

Fig. 2

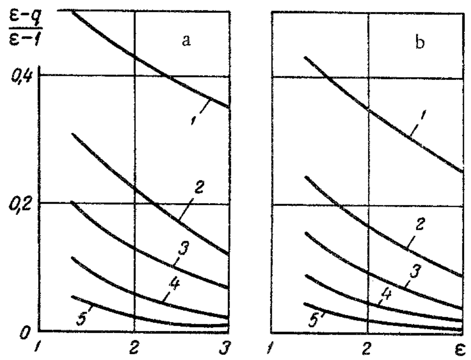

Fig. 3

Fig. 2. Maximum thermal flux $\overline{q}$ over the inside surface of a tube, at the groove center, as a function of the length of the grooved segment l. a) $r_0 = 9$; b) $r_0 = 1$: 1) $\varepsilon = 1.35$; 2) 1.5; 3) 2.0; 4) 3.0.

Fig. 3. Relative deviation of the maximum thermal flux over a groove $(\varepsilon - \overline{q})/(\varepsilon - 1)$, as a function of the groove depth parameter ε. a) $r_0 = 1$; b) $r_0 = 9$: 1) $l = 0.5$; 2) 1.0; 3) 1.5; 4) 2.25; 5) 4.0.

The difference between the maximum value of thermal flux over a groove and its asymptotic value $(\varepsilon - \overline{q})/(\varepsilon - 1)$ decreases as the groove depth parameter increases (Fig. 3).

These calculations show that, when analyzing heat-transfer experiments for a study of local short risers, it is necessary to take into account the lengthwise heat leakage from the wall of the test segment.

NOTATION

a is the coefficient;
2L is the groove width;
$2l$ is the dimensionless groove length;
R is the radius;
r is the dimensionless radius;
q_v is the density of volume heat sources;
T is the temperature;
u_i are the eigenfunctions;
z is the coordinate along tube axis;
Z is the dimensionless coordinate along tube axis;
ε is the groove depth parameter;
λ is the thermal conductivity of tube wall material;
μ are the eigenvalues;
θ is the dimensionless temperature.

LITERATURE CITED

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